On the Performance of MIMO-NOMA-Based Visible Light Communication Systems

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Abstract—In this letter, we apply the non-orthogonal multiple access (NOMA) technique to improve the achievable sum rate of multiple-input multiple-output (MIMO)-based multi-user visible light communication (VLC) systems. To ensure efficient and low-complexity power allocation in indoor MIMO-NOMA-based VLC systems, a normalized gain difference power allocation (NGDPA) method is first proposed by exploiting users' channel conditions. We investigate the performance of an indoor 2 x 2 MIMO-NOMA-based multi-user VLC system through numerical simulations. The obtained results show that the achievable sum rate of the 2 x 2 MIMO-VLC system can be significantly improved by employing NOMA with the proposed NGDPA method. It is demonstrated that NOMA with NGDPA achieves a sum rate improvement of up to 29.1% compared with NOMA with the gain ratio power allocation method in the 2 x 2 MIMO-VLC system with three users.

Index Terms—Visible light communication (VLC), multiple-input multiple-output (MIMO), non-orthogonal multiple access (NOMA), orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Due to its many inherent advantages such as license-free spectrum, low-cost front-ends, high security and strong immunity to electromagnetic interference, white light-emitting diodes (LEDs) enabled visible light communication (VLC) has attracted considerable attention for high-speed and short-range wireless communications recently [1]. The main challenge to develop large-capacity VLC systems is the small modulation bandwidth of off-the-shelf LEDs. So far, a few techniques have been proposed to boost the capacity of VLC systems, such as orthogonal frequency division multiplexing (OFDM), multiple-input multiple-output (MIMO) and so on [2], [3].

In practical VLC systems, one LED transmitter is generally expected to support multiple users. Therefore, multiple access is essential in the multi-user VLC systems. In [4], orthogonal frequency division multiple access (OFDMA) has been applied in VLC systems. However, the achievable data rate employing OFDMA is inevitably reduced due to the spectrum partitioning. Recently, non-orthogonal multiple access (NOMA) via power domain multiplexing has been proposed for 5G systems due to its superior spectral efficiency [5]. In NOMA systems, all users can use the entire modulation bandwidth of the system through power domain superposition coding at the transmitter side and successive interference cancellation (SIC) at the receiver side. It has been shown that NOMA performs much better in high signal-to-noise ratio (SNR) scenarios [6]. Considering the fact that VLC systems offer high SNRs due to the short distance between the transmitter and the receiver, it is beneficial to apply NOMA in downlink VLC systems. The performance of NOMA-based VLC systems has been widely investigated [7]–[10]. In [7], the authors suggested NOMA as a potential candidate for high-speed VLC systems and also proposed a gain ratio power allocation (GRPA) method. More advanced power allocation methods for NOMA-VLC were reported in [8] and [9], but with relatively high computational complexity. In [10], a phase pre-distortion approach was proposed to improve the error rate performance of uplink NOMA-VLC systems.

As a natural and efficient way to increase system capacity and extend system coverage, MIMO has been widely applied in VLC systems by exploiting illuminating LED arrays [2], [11]. However, the application of NOMA in MIMO-VLC systems has been barely investigated. In [12], a MIMO-NOMA-based VLC system was experimentally verified, but power allocation was not considered. When applying MIMO, power allocation methods of single-LED NOMA-VLC systems cannot be directly adopted in MIMO-NOMA-based VLC systems. So far, several power allocation methods have been proposed in literature for MIMO-NOMA radio frequency (RF) systems, such as hybrid precoding and post-detection [13] and signal alignment [14]. However, these methods have high computational complexity. In practical VLC systems, efficient power allocation methods with low computational complexity are of vital importance for the potential wide application of the MIMO-NOMA technique.

In this letter, we apply NOMA in MIMO-VLC systems and propose a novel power allocation method, i.e. normalized gain difference power allocation (NGDPA), for efficient and low-complexity power allocation in MIMO-NOMA-VLC systems. The sum rate performance of an indoor 2 x 2 MIMO-NOMA-VLC system is evaluated via numerical simulations. It is shown that the achievable sum rate of the 2 x 2 MIMO-VLC system can be greatly improved by employing NOMA with the proposed NGDPA method in comparison to NOMA with GRPA.
for each LED, the following power constraint is imposed:

\[ P_{elec} = \sum_{k=1}^{K} \rho_{i,k} = P_{elec}. \]  

(2)

Without loss of generality, we assume that \( P_{elec} = 1 \). After free-space propagation, the received electrical signal vector at the \( k \)-th user is given by

\[ y_k = \gamma P_{opt} \xi H_k x + n_k, \]  

(3)

where \( \gamma \) is the responsivity of the PD, \( P_{opt} \) is the output optical power of the LED, \( \xi \) is the modulation index, \( H_k \) is the 2x2 channel matrix of the \( k \)-th user, \( x = [x_1 \ x_2]^T \) is the transmitted electrical signal vector where \( [\cdot]^T \) is the transpose operation, and \( n_k \) is the additive noise vector. The detailed calculation of the variances of the additive noises can be found in [3].

Assuming each LED follows a Lambertian radiation pattern and only the line-of-sight (LOS) component is considered, the LOS optical channel gain between the \( i \)-th LED and the \( j \)-th PD of the \( k \)-th user is calculated by [11]

\[ h_{ji,k} = \frac{(m + 1) A_{PD}}{2\pi d^2} \mu \eta \cos^m(\varphi) \cos(\theta), \]  

(4)

where \( m = -\ln(2)/\ln(\Phi) \) is the Lambertian emission order and \( \Phi \) is the semi-angle at half power of the LED, \( A_{PD} \) is the active area of the PD, \( d \) is the distance between the \( i \)-th LED and the \( j \)-th PD, \( \mu \) and \( \eta \) are the gains of the optical filter and optical lens, respectively, \( \varphi \) is the emission angle, and \( \theta \) is the incident angle. Note that the optical channel gain becomes zero if the incident light is outside the field-of-view (FOV) of the receiver.

In order to successfully recover the transmitted data, MIMO de-multiplexing is performed and zero-forcing (ZF) using basic channel inversion is used due to its low complexity [3]. After ZF-based MIMO de-multiplexing and power normalization, the estimated electrical signal vector at the \( k \)-th user is obtained by

\[ \hat{x}_k = x + \frac{1}{\gamma P_{opt} \xi} H_k^{-1} n_k, \]  

(5)

where \( H_k^{-1} \) is the inverse of \( H_k \). Subsequently, SIC is performed with respect to each LED at each user. To perform SIC, the decoding order of the users with respect to each LED should be first determined. Differing from single-LED NOMA, multiple LEDs are involved in MIMO-NOMA. Hence, a new way to determine the decoding order should be developed. Instead of individual optical channel gains of each user adopted in single-LED NOMA-VLC systems [7], we use the sum of the optical channel gains of each user with respect to each LED to sort the users. Without loss of generality, assuming that \( K \) users with respect to the \( i \)-th LED are sorted according to their sum optical channel gains in the decreasing order as follows:

\[ h_{1i,1} + h_{1i,2} + \cdots + h_{1i,K} + h_{2i,1} + h_{2i,2} + \cdots + h_{2i,K}, \]  

(6)

the decoding order with respect to the \( i \)-th LED is then set to be:

\[ O_{i,1} < O_{i,2} < \cdots < O_{i,K}. \]  

(7)

The detailed procedures of SIC can be found in [7]. Here, we assume that perfect SIC can be performed without signal error propagations [9]. Finally, the data for two users are obtained via DCO-OFDM demodulation.

III. Power Allocation Methods

Power allocation plays an important role in NOMA systems [8], which is also one of the key challenges to apply the NOMA technique in cost-sensitive MIMO-VLC systems. In this work, two low-complexity power allocation methods are introduced for MIMO-NOMA-VLC systems. The main focus of this work is to evaluate the achievable sum rate of MIMO-NOMA-based VLC systems using low-complexity power allocation methods. To guarantee user fairness, user pairing or grouping techniques can be incorporated, which is beyond the scope of this letter.

A. Gain Ratio Power Allocation (GRPA)

As a simple but efficient power allocation method, GRPA has been proposed for single-LED NOMA-VLC systems [7]. In GRPA, power allocation depends on the optical channel
gain of each user. By replacing the optical channel gain with the sum optical channel gain, the GRPA method can be generalized for MIMO-NOMA-based VLC systems. In the $2 \times 2$ MIMO-NOMA-VLC system using GRPA, according to the decoding order in (7), the relationship between the electrical powers allocated to user $k$ and user $k+1$ in the $i$-th LED is described by

$$\rho_{i,k} = \left( \frac{h_{1i,k+1} + h_{2i,k+1}}{h_{1i,1} + h_{2i,1}} \right)^{k+1} \rho_{i,k+1}. \quad (8)$$

**B. Normalized Gain Difference Power Allocation (NGDPA)**

To further improve the achievable sum rate of VLC systems employing MIMO-NOMA, we propose an efficient and low-complexity NGDPA method for MIMO-NOMA-VLC systems. By exploiting the optical channel gain difference, the electrical powers allocated to user $k$ and user $k+1$ in the $i$-th LED have the following relationship:

$$\rho_{i,k} = \left( \frac{h_{1i,1} + h_{2i,1} - h_{1i,k+1} - h_{2i,k+1}}{h_{1i,1} + h_{2i,1}} \right)^{k} \rho_{i,k+1}. \quad (9)$$

As we can see from (9), optical channel gain difference is used instead of the absolute values of the optical channel gains as in (8), and the power of the ratio is reduced from $k+1$ to $k$.

**IV. SIMULATION RESULTS**

In this section, we evaluate the performance of a $2 \times 2$ MIMO-NOMA-VLC system using different power allocation methods through numerical simulations. The geometric setup of the $2 \times 2$ MIMO-NOMA-VLC system is illustrated in Fig. 1, where the spacing between two LEDs is 1 m and the vertical distance between the LEDs and the users is 2.15 m. The semi-angle at half power and the output optical power of each LED are $60^\circ$ and $10$ W, respectively. The modulation index is angle at half power and the output optical power of each user is $4$ cm. The gain of the optical filter is $0.9$ and the gain of the optical lens is $2.5$. As shown in Fig. 1, user 1 is assumed to face at the center between LED 1 and LED 2, and its location is fixed. The distance between user 1’s location and the edge of the system coverage is $R = 2$ m. All the $K$ users are uniformly distributed and the distance between user 1 and user $K$ is given by $r$. We define the normalized offset of user $K$ with respect to user 1 as $r/R$, and the normalized offset of user $k$ with respect to user 1 is given by $(k-1)r/R$. For performance comparison, we also study OFDMA with equal power allocation in the system [9].

Fig. 3 shows the achievable bit rate of each LED versus the normalized offset $r/R$ in the $2 \times 2$ MIMO-NOMA-VLC system with two users ($K = 2$), where the achievable rate of each LED is the aggregated achievable rate of two users served by the LED. It can be seen that the achievable rate of LED 1 remains stable at about $56.5$ Mbit/s when $0.1 \leq r/R \leq 0.6$ for both GRPA and NGDPA. However, the achievable rate of LED 1 using GRPA is greatly reduced when $r/R$ is further increased and it is only about $36.9$ Mbit/s at $r/R = 1$.

In contrast, the achievable rate of LED 1 utilizing the proposed NGDPA is only slightly reduced with the further increase of $r/R$ and a bit rate of $51.9$ Mbit/s can be achieved at $r/R = 1$. As can also be seen, the achievable rate of LED 2 is almost the same at different $r/R$ values for GRPA and NGDPA. Moreover, the achievable rate of LED 2 suffers from certain fluctuation with the increase of $r/R$ and a minimum rate of $49.4$ Mbit/s is achieved at $r/R = 0.5$, for both GRPA and NGDPA. This rate reduction can be explained as follows: with respect to LED 2, two users have the same channel conditions at $r/R = 0.5$ and hence, the achievable rate is low due to the poor performance of NOMA. Although user 2 has the same channel conditions with respect to LED 2 for $r/R = 0.1$ and $0.4$ due to geometric symmetry, the achievable rate of LED 2 is reduced when $r/R$ is increased from 0.1 to 0.4. This is because the SNR of user...
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2 with respect to LED 2 is reduced when \( r/R \) is increased from 0.1 to 0.4, which leads to the reduced achievable rate. 

Fig. 4 shows the achievable sum rate versus \( r/R \) in the \( 2 \times 2 \) MIMO-VLC system employing OFDMA and NOMA. As can be seen, the sum rate utilizing OFDMA is continuously reduced with the increase of \( r/R \). By applying NOMA with GRPA, the sum rate is increased for both \( K = 2 \) and 3. When \( K = 2 \), the sum rate fluctuates with the increase of \( r/R \) and a relatively low sum rate of about 106.4 Mbit/s is achieved at \( r/R = 0.5 \), due to rate reduction of LED 2 at \( r/R = 0.5 \) as explained above. When \( K = 3 \), the sum rate is stable as \( 0.1 \leq r/R \leq 0.6 \). For both \( K = 2 \) and 3, a fast decrease occurs when \( r/R > 0.6 \), indicating that NOMA with GRPA suffers from significant sum rate loss when \( r/R \) becomes relatively large. For \( 0.7 \leq r/R \leq 1 \), the achievable sum rate using NOMA with GRPA is reduced when \( K \) is increased from 2 to 3. However, when NOMA with NGDPA is used, the achievable sum rate is substantially improved compared with NOMA with GRPA, especially when \( r/R > 0.7 \). It also can be observed that similar sum rate performance can be achieved for \( K = 2 \) and 3 when using NOMA with the proposed NGDPA method. 

Fig. 5 shows the sum rate gain of NGDPA over GRPA versus \( r/R \). It can be found that the sum rate gain becomes significant when \( r/R \) is relatively large. Furthermore, the sum rate gain is much more significant when the number of users is increased. For example, when \( r/R = 1 \), i.e. user \( K \) is located at the edge of the system coverage, the achieved sum rate gains are 16.7% and 29.1% for \( K = 2 \) and 3, respectively. 

V. Conclusion

In this letter, a novel NGDPA method exploiting the users’ channel conditions has been proposed for adaptive and efficient power allocation in indoor MIMO-NOMA-VLC systems with low computational complexity. It is shown by simulation results that, in an indoor \( 2 \times 2 \) MIMO-VLC system, NOMA with NGDPA achieves greatly improved sum rate than NOMA with GRPA. Up to 29.1% sum rate improvement can be achieved by utilizing NOMA with NGDPA in the \( 2 \times 2 \) MIMO-VLC system with three users. It is also verified that similar sum rate can be achieved with two or three users in the system by using NOMA with NGDPA. Furthermore, the sum rate gain of NGDPA over GRPA is much more significant with more users in the system. Therefore, MIMO-NOMA with the proposed NGDPA method is promising for future high-speed multi-user VLC systems.

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REFERENCES